Perspectives on the application of RVT in ground motion modeling, soil response, and structural dynamics

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Random vibration theory summary

- Uses extreme value statistics to predict peak time values in the time domain using frequency content and duration
- Permits moving between Fourier spectrum and response spectrum without time series; eliminates the need of time series for analyses
- Unlocks the potential to develop Fourier-domain models and easily convert to response spectra for engineering application
- *Adoption of Fourier-domain models permits application of small-magnitude data to develop and calibrate models*

Outline

Introduction to RVT

Benefits of Fourier-domain modeling

Example applications

Future advances

Conclusions

Introduction to RVT

Introduction to RVT

- Random-vibration theory statistically represents a time series
- Allows calculation of the *expected* time-domain peak with one calculation (i.e., PGA) using extreme value statistical models
- Eliminates the need for multiple time series with different phasing, which might required 10-20 motions for a stable estimate of the mean
- Input motion can be modified by frequency domain response to provide SDOF effects (response spectrum), site response, and structural response
- Assumes stationarity (characteristics don't change with time); not the case but method still works
- More details provided in Boore (2003), Houtte, Larkin, and Holden (2018), Kottke et al. (2021), and Wang and Rathje (2018a)

RVT motion description

- Seismology typically works on Fourier spectra – power spectra also used
- Parseval's theorem, equal energy in time and frequency domains:

$$x_{\text{rms}} = \sqrt{\frac{2}{D_{\text{rms}}} \int_0^\infty |X(f)|^2 \, df} = \sqrt{\frac{m_0}{D_{\text{rms}}}}$$

- Spectrum is quantified by spectral moments:

$$m_n = 2 \int_0^\infty (2\pi f)^n |X(f)|^2 \, df$$

m_0, m_1, m_2, and m_4 are typically used.

Peak value calculation

- Peak factors are used to relate root-mean-squared (x_{rms}) amplitude to peak amplitude (x_{max})
- Cartwright and Longuet-Higgins (1956, CLH56) considers a stationary response over a time interval:

$$\mathrm{PF} = \frac{x_{\mathrm{max}}}{x_{\mathrm{rms}}} = \sqrt{2}\int_0^{\infty} \left\{1 - \left[1 - \varepsilon \exp(-z^2)\right]^{N_e}\right\} dz$$

where

$$\varepsilon = \frac{N_z}{N_e} = \sqrt{\frac{(m_2)^2}{m_0 \cdot m_4}} \quad N_z = \frac{1}{\pi}\sqrt{\frac{m_2}{m_0}} \cdot D_{\mathrm{gm}} \quad N_e = \frac{1}{\pi}\sqrt{\frac{m_4}{m_2}} \cdot D_{\mathrm{gm}}$$

- CLH56 has been used for almost 40 years within the seismology community

Vanmarcke (76) peak factors

- Vanmarcke (1976) includes potential for clumping and no longer assumes statistically independent peaks:

$$F_x(x) = \left[1 - \exp\left(-\frac{x^2}{2}\right)\right] \cdot \exp\left\{-\frac{N_z\left[1 - \exp\left(-\sqrt{\pi/x}\cdot\delta_e\cdot x\right)\right]}{1 - \exp\left(-x^2/2\right)}\right\}$$

  where δ_e is an empirical factor defined as:

$$\delta_e = \delta^{1+b} \qquad \delta = \sqrt{1 - \frac{(m_1)^2}{m_0 \cdot m_2}}$$

  and b was estimated as 0.2 from numerical simulations
- Simplified to closed-form solution by Kiureghian (1980)
- Wang and Rathje (2018a,b) found improved performance in site response applications
- Ongoing re-evaluation by PG&E and LCI of non-stationarity modification

Ground-motion duration for RVT calculation

Duration defined by either:

1. Significant duration; Kolli and Bora (2021) found D_{5-75} performs best and consist with recommendations from Vanmarcke and Lai (1980) and Ou and Herrmann (1990) – defines the stationary part of the signal
2. Compatible duration; duration to best-fit time domain intensity initially proposed by Bora et al. (2014)

Challenges associated with RVT

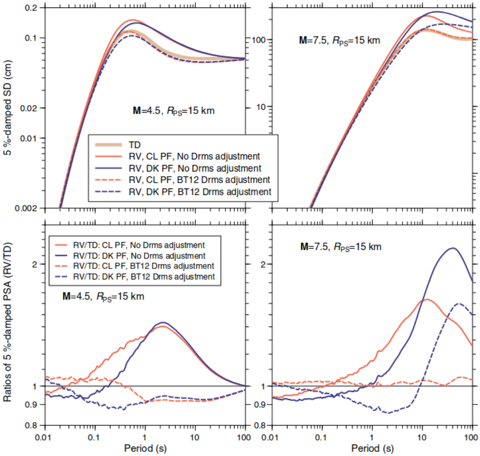


Figure 6. (Top) Response spectral displacements (SD) from time-domain (TD) simulations and random-vibration (RV) simulations, without and with adjustments for the duration (D_{rms}) used to compute the root mean square (rms) of the oscillator response. The D_{rms} adjustments are those of Boore and Thompson (2012; BT12), not those of this study. The Boatwright and Seekins (2011) attenuation model, the path durations in this study, and a stress parameter of 275 bars were used in the simulations. For the RV simulations, two rms-to-peak factors were used: CL (Cartwright and Longuet-Higgins, 1956) and DK (Der Kiureghian, 1980). (Bottom) Ratios of the RV and TD response spectra. The color version of this figure is available only in the electronic edition.

From Boore and Thompson (2015)

Challenges associated with RVT

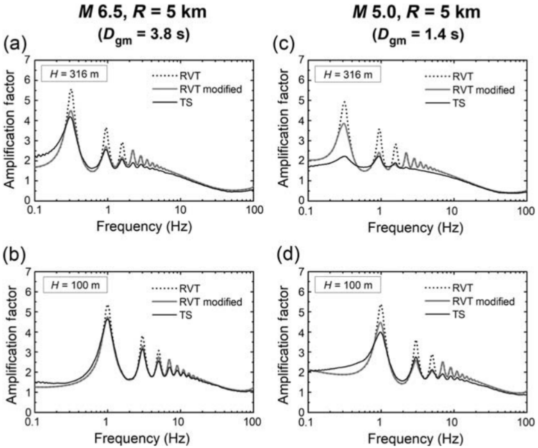


Figure 3. Amplification factor (AF) results from linear-elastic (LE) RVT and time-series (TS) analyses for (a,c) $H = 316$ m and (b,d) $H = 100$ m hypothetical sites with $V_{S,\mathrm{rock}} = 3000$ m/s and selected earthquake scenarios. Results shown for RVT analysis, RVT analysis with the duration modification, and TS analysis.

From Wang and Rathje (2018b)

Challenges associated with RVT

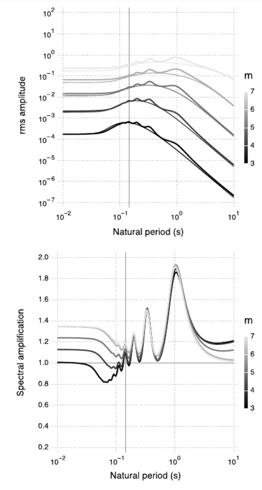
- Underlying assumptions of RVT are inconsistent with observed earthquake recordings
- Duration been used to correct peak factor formulations for improved performance: Boore and Joyner (1984), Boore and Thompson (2015), and Liu and Pezeshk (1999)

  Fixes the problem, but isn't robust
- For relative calculations or specifying an RVT motion based on a response spectrum then misfit becomes less important

Benefits of Fourier-domain modeling

Response spectrum characteristics

- Invaluable intensity measure from an engineering perspective
- Unfortunately, oscillator response depends on response at lower frequencies
- Predictive model components must include this magnitude dependence



Stafford, Rodriguez-Marek, et al. (2017)

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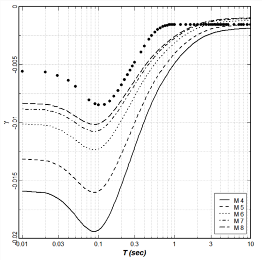


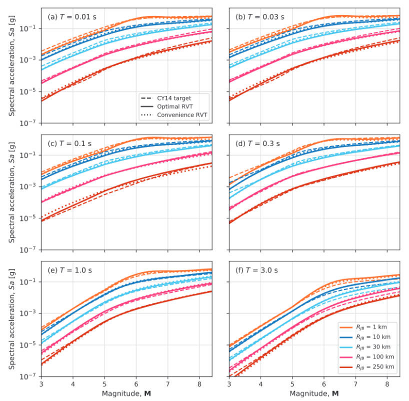
Chiou and Youngs (2008)

Fourier amplitude characteristics

- Not directly usable for engineering purposes
- Response at a given frequency is independent of other frequencies
- Source, path, site, and structural effects are applied as linear filters
- Permits leveraging small-magnitude data to develop models; critical for development of non-ergodic models

Comparison of SA and FAS models

- Chiou and Youngs (2014, CY14) NGA-West2 ground motion model defined at 24+2 periods by 22 (period dependent) & 6 (period independent) for a total of **578** coefficients
- Stafford, Boore, et al. (2022) point-source model developed to fit CY14 model and uses only **14** parameters
- Increased are partly due to complexity of response spectrum



Example applications

RVT tools

`pyRVT` Python implementation of peak factors
`https://github.com/arkottke/pyrvt`

- Peak factor models: Vanmarcke (1975), Davenport (1964), Der Kiureghian (1985), Toro & McGuire (1987), Cartwright & Longuet-Higgins (1956), Boore & Joyner (1984), Liu & Pezeshk (1999), Boore & Thompson (2012), Boore & Thompson (2015), and Wang & Rathje (2018)
- Point-source models and response spectrum compatible RVT motions
- Used for Atik et al. (2014)

`pySRA` Site response with time series and RVT using `pyRVT`
`https://github.com/arkottke/pysra`

Complex site response calculations

From a not-so-theoretical project:

- 5 sites
- ≈6,000 alternatives profiles per site
- 60 simulated profiles per profile
- Input motions defined by conditional mean spectra with 3 conditioning periods, 8 intensity levels, and 4 magnitudes
- 172,800,000 site response calculations (at 1 calculation per second: 5.5 years CPU time)

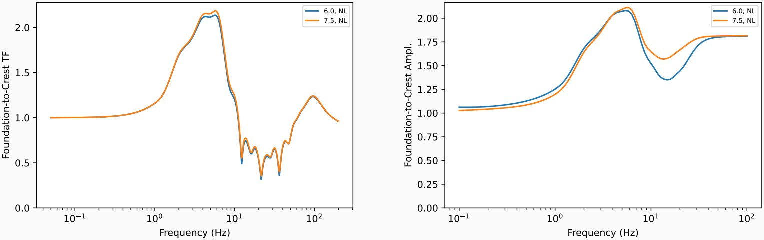
Using time series would require:

- Selection and modification of time series for each of the 96 input scenarios
- Representative time series might not be available
- Assuming 11 time series, need 11x more calculations

Dam foundation-to-crest amplification

- **Goal:** Compute acceleration response spectra at the crest of dam.
- No simplified model; instead could be achieved through nonlinear FEM analyses
- Use existing models and RVT:
  1. PG&E has developed model for dam period based on ambient noise recordings
  2. Park and Kishida (2019) developed empirical models based on foundation and crest recordings to predict natural period and PGA_{crest}
  3. Gazetas and Dakoulas (1992) analytical solutions modal response; used to construct transfer functions
  4. Damping and velocity in the Gazetas and Dakoulas (1992) model adjusted to fit nonlinear suggested by Park and Kishida (2019) model.

Foundation-to-crest amplification



- PGA$_{\text{foundation}}$ same for both scenarios
- Similar Fourier amplification, but different response spectral amplification
- Magnitude dependence due to differences in spectral shape

Future advances

Future advances in RVT application

- Theoretical peak factor models should be tuned to improve agreement ground-motion recordings:
    - Oscillator corrections (e.g., Boore and Thompson 2015) provide direct correction, but have limited used when coupled with additional response (e.g., site or structural response)
    - Better to incorporate theoretically-consistent corrections
- Use peak factor parameters to develop new peak value predictions:
    - Combine theoretical behavior with observations
    - N. Abrahamson currently working on this approach
    - Based on the observation peak-factor models do not perform well for near-fault ground motions

Conclusions

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